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TECHNICAL NOTES

Combined thermal–momentum start-up in long pipes

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1. INTRODUCTION

THE FLOW regulation of fluid systems involving transient laminar forced convection has become important in connection with high performance heat transfer equipment requiring precision control. Previous mathematical and experimental investigations have been limited mostly to the cases of steady flow with changes in wall heat flux, inlet temperature or wall temperature [1–7]. The problem concerning combined thermal and momentum start-up has been given little attention. Creff and Andre [8] provided numerical solutions for certain flow conditions including a pulsating flow situation.

In the combined thermal–momentum start-up problem, both the velocity and temperature profiles change with time. The solution of the combined thermal–momentum problem is complicated mathematically due to coupling of the partial differential equations of energy, momentum and continuity. In the approach used here for the constant-property combined thermal–momentum start-up in long pipes, the momentum equation is decoupled from the energy equation. Szymanski's [9] analytical velocity profile for the momentum equation is substituted into the energy equation which is solved numerically for the combined thermal–momentum start-up. The results for the variation of Nusselt number and average temperature are presented at various dimensionless times. A thermal start-up time parameter ($Fo_{0.99}$)—defined in terms of Fourier number at which the average fluid temperature reaches 99% of the corresponding steady state value—is introduced to qualify the transient thermal flow development along the pipe length. Finally, the numerical results are correlated to provide an estimation procedure for the start-up time.

2. MATHEMATICAL FORMULATION AND SOLUTION PROCEDURE

Consider a long circular pipe of radius R that is filled initially (i.e. $t = 0$) with a constant-property, incompressible, Newtonian fluid at a constant temperature T_0 throughout. At $t > 0$, the fluid is exposed to a constant pressure gradient $(-\Delta P/L)$ in the axial direction while a step change in temperature is simultaneously imposed at the wall ($r = R, 0 < z$). Note that there are no entrance effects for flow development in long pipes, hence $u \equiv u(r, t)$.

The equations governing the heat and momentum transfer in axisymmetric transient laminar flow of a constant-property, incompressible, Newtonian fluid in a long horizontal straight pipe are

$$\rho C\left[\frac{\partial}{\partial t}T(r,z,t)+u(r,t)\frac{\partial}{\partial z}T(r,z,t)\right] = \frac{k}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}T(r,z,t)\right] \quad (1)$$

$$\rho\frac{\partial}{\partial t}u(r,t) = \frac{-\Delta P}{L}+\frac{\mu}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}u(r,t)\right]. \quad (2)$$

The initial conditions for the combined thermal–momentum start-up problem are

$$T(r,z,0) = T_0; \quad 0 < z, \quad 0 < r < R \quad (3)$$

$$u(r,0) = 0; \quad 0 < r < R. \quad (4)$$

The boundary conditions are

$$u(R,t) = 0; \quad 0 < t \quad (5)$$

$$\frac{\partial}{\partial r}u(0,t) = 0; \quad 0 < t \quad (6)$$

$$T(r,0,t) = T_0; \quad 0 < r < R, \quad 0 < t \quad (7)$$

$$T(R,z,t) = T_w; \quad 0 < z, \quad 0 < t \quad (8)$$

$$\frac{\partial}{\partial r}T(0,z,t) = 0; \quad 0 < z, \quad 0 < t. \quad (9)$$

Since the fluid properties are assumed to be independent of temperature, the momentum equation is not coupled to the energy equation and therefore may be solved analytically [4]. The following velocity distribution for flow start-up in long pipes was given by Szymanski [9]

$$\phi = (1-\xi^2)-8\sum_{n=1}^{\infty}\frac{J_0(\alpha_n\xi)}{\alpha_n^3 J_1(\alpha_n)}e^{-\alpha_n^2\tau} \quad (10)$$

where α_n are the positive roots of $J_0(\alpha_n)$. The terms ϕ, ξ and τ in equation (10) are

$$\phi = \frac{u}{-\Delta PR^2/4\mu L}; \quad \xi = \frac{r}{R}; \quad \tau = \frac{\mu t}{\rho R^2}. \quad (11)$$

Note that the pressure gradient $(-\Delta P/L)$ should not be greater than 8000 $\mu^2/\rho R^3$ in order for the flow to be laminar (i.e. $Re < 2000$). The use of equation (10) as the solution to equation (2) simplifies the combined thermal–momentum start-up problem to a single partial differential equation (i.e. equation (1)). Equation (1) was expressed in finite difference form and the resulting set of non-linear equations was solved numerically using a Newton–Raphson convergence scheme. The details of grid distribution and numerical technique are presented elsewhere [10, 11].

3. RESULTS AND DISCUSSION

For comparison, two steady-flow thermal-entry problems involving a step change in wall temperature are considered.

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NOMENCLATURE

C	specific heat [J kg^{-1} K^{-1}]
Fo	Fourier number, $kt/\rho CR^2$
$Fo_{0.99}$	thermal start-up time parameter
h	local heat transfer coefficient $k(\partial T/\partial r)_w/(T_w-\bar{T})$ [W m^{-2} K^{-1}]
J_0, J_1	Bessel function of the first kind of order 0, 1
k	thermal conductivity [J m^{-1} K^{-1}]
Nu	local Nusselt number, $2hR/k$
Pr	Prandtl number, $\mu C/k$
$-\Delta P/L$	pressure gradient [Pa m^{-1}]
R	pipe radius [m]
r	radial distance [m]
Re	Reynolds number, $2\rho\bar{u}R/\mu$
T	temperature [K]
$\bar{T}$	bulk temperature [K]
T_0	initial and inlet temperature [K]
T_w	wall temperature [K]
t	time [s]
u	axial velocity [m s^{-1}]
$\bar{u}$	average axial velocity [m s^{-1}]
X^+	dimensionless distance, $(z/R)/(Re\,Pr)$
z	axial distance [m].

Greek symbols

α_n	positive roots of $J_0(\alpha_n)$
θ	dimensionless temperature, $(\bar{T}-T_w)/(T_0-T_w)$
μ	dynamic viscosity [Pa s]
ξ	dimensionless radial distance, r/R
ρ	density [kg m^{-3}]
τ	dimensionless time, $\mu t/\rho R^2$
ϕ	velocity distribution parameter, $4u\mu L/(-\Delta P)R^2$.

The first is the well-known Graetz problem concerning steady state laminar forced convection in circular pipes. The second problem examines the transient behaviour of the Graetz problem, for which a solution was proposed by Cotta and Ozisik [5]. The solutions for these two problems are compared in Fig. 1. For short X^+, the dimensionless temperature for the transient problem approaches the Graetz solution almost immediately. However, the start-up time is seen to increase with X^+. At $X^+ = 0.1$, for example, the thermal start-up is accomplished at $Fo < 0.2$.

The calculated temperature profiles for the combined thermal–momentum start-up problem are compared with the Graetz solution in Fig. 2. The results are markedly different from those in Fig. 1 for the transient Graetz problem. At early times (i.e. small Fo), conduction as opposed to forced convection is the predominant mode of heat transfer because the fluid velocity is quite small. Hence, the thermal state along the pipe length is nearly uniform. As the velocity development proceeds with time, the relative magnitude of the convective mode of heat transfer increases. At $Fo = 0.2$, the temperature profile is far from the steady state Graetz solution but is beginning to approach it.

The calculated Nusselt number values for the combined thermal–momentum start-up problem are plotted, and compared with the Graetz solution, in Fig. 3. At very small X^+, the Nusselt number increases with an increase in Fourier number. But at large X^+, Nu decreases with time from an initially larger value to approach the Graetz solution.

To quantify the difference between the start-up times for the transient Graetz problem and the combined thermal–momentum problem, a thermal start-up time parameter, $Fo_{0.99}$, is defined. For a given X^+, $Fo_{0.99}$ gives the time at which the dimensionless temperature θ is 99% of the corresponding Graetz solution. The start-up time parameters for the transient Graetz and the combined thermal–momentum problems are compared in Fig. 4. Clearly, $Fo_{0.99}$ for the combined thermal–momentum problem is much larger than that for the transient Graetz problem. This seems reasonable in view of the fact that the mode of heat transfer changes from conduction to forced convection in the combined thermal–momentum problem as opposed to forced convection throughout for the transient Graetz problem. Based on

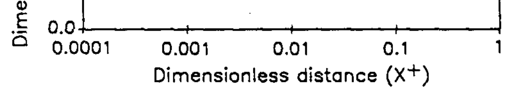

FIG. 1. Temperature profiles for the transient and steady state cases of the Graetz problem.

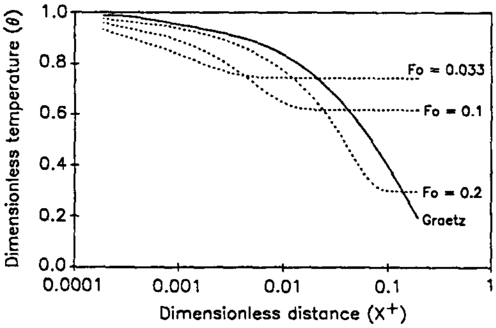

FIG. 2. Development of the temperature profile for the combined thermal–momentum start-up problem.

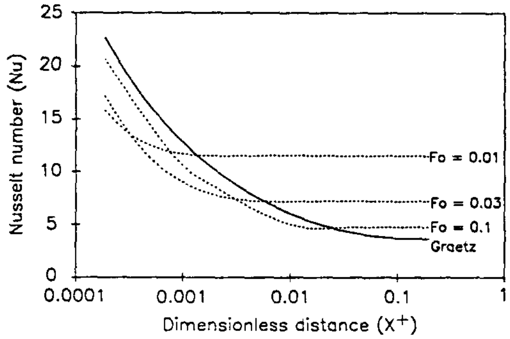

FIG. 3. Calculated Nusselt number for the combined thermal–momentum start-up problem.

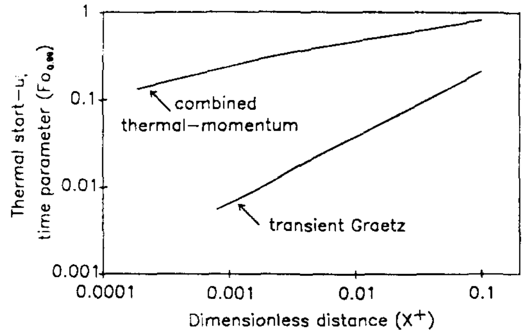

FIG. 4. Thermal start-up times for the transient Graetz and the combined thermal–momentum start-up problems.

the results presented in Fig. 4, equations (12) and (13) are proposed for estimating the start-up time in the transient Graetz and the combined thermal–momentum problems, respectively:

transient Graetz

$$Fo_{0.99} = 1.2(X^+)^{3/4}; \quad (12)$$

combined thermal–momentum

$$Fo_{0.99} = 1.5(X^+)^{1/4}. \quad (13)$$

4. CONCLUSIONS

A solution method is outlined for the combined thermal-momentum start-up problem. By assuming the fluid properties to be constant, the momentum equation is decoupled from the energy equation. The energy equation is expressed in a finite difference form and solved numerically with the velocity profile calculated from the analytical solution of the momentum equation. Results indicate that the start-up time for the combined thermal–momentum start-up problem is much larger than that for the transient Graetz problem. Simple correlations are proposed for the start-up time for the two transient laminar heat transfer problems.

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Moisture and temperature dependence of the moisture diffusivity

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THE DEPENDENCE of the moisture diffusivity $D(u, T)$ on the moisture content u and on the temperature T is important not only in technical applications of porous materials, but it can also yield valuable information on the mechanism of the unsaturated water flow in capillary-porous media. It is usually assumed [1] that in the transport of the liquid phase the surface tension σ plays the role of the driving force and the viscosity η of the liquid is responsible for energy dissipation that leads to a quasi-stationary character of water flow. Dimensional analysis yields the following expression